Report SMiLES to Qbuzz

Auriane CADORIN

December 23, 2021

Abstract

Qbuzz, as a large public transport company operating in the Netherlands, wants to reduce its carbon emissions. They are already very low, indeed Qbuzz has a lot of electric buses, functioning with green electricity or hydrogen, and the other buses use HVO biodiesel. Their objective for the next decades is to go to 100% of electric buses but the cost of green electricity is rising and the cost hydrogen is very high. This is why they want to find a way to optimize their consumption of electricity to achieve their goal. After a few conversations with them, we thought about a few solutions that we will try to develop here. The first one is to reduce the costs to the root : most of the green electricity in the Netherlands come from Wind Power Producers (WPPs), who have to deal with the uncertainty of the wind which impacts the price of their services. To help them reduce these costs, we can show that a coalition of WPPs from all the Netherlands would decrease this uncertainty and that instead of being in competition they could become one entity, and therefore be more profitable. A second option would be to create a Balance Responsible Party (BRP). A BRP is a society whose clients are producers or consumers of electricity, its goal is to balance the consumption and production of electricity. For example if they realize they will have too much electricity a few days in advance, they can encourage their consumers clients to consume more electricity on that day for a lower price or sell it to another BRP that does not have enough electricity. In our example, if Qbuzz was part of the BRP, the company would have a more direct vision of the electricity production in the BRP so on days where too much electricity is produced, Qbuzz could buy more electricity and store it. This leads us to the third solution : store electricity on low peak demand. On this low demand peaks, electricity price is very low cause producers cannot store it and need to sell it if they do not want to pay extra fees for disturbing the grid. We discussed about the different ways Qbuzz could store electricity and they already had a few ideas. Indeed the batteries of the buses will have to be changed after a decade because they will not perform enough anymore. Instead of throwing them away, Qbuzz thinks about building an electricity storage center. They could fill the batteries up on low demand peaks and use the electricity on high demand peaks, when the price of energy is the most expensive. As Groningen is apart of the Hydrogen Valley and Qbuzz has hydrogen buses, the company also thought about creating its own hydrogen on low demand peaks.

1 Coalition of WPPs

In this section, we will introduce the general formulation of our problem, which is to prove that the WPPs would make more money as a coalition than alone.

We consider a collection of independent wind power producers (WPPs) indexed by $i \in \mathcal{N} := \{1, 2, ..., n\}$. The wind power produced by producer $i \in \mathcal{N}$ is modeled as a scalar valued stochastic process $w_i(t) \in [0, W_i]$, where W_i is the capacity of WPP i. We define the wind power processes on the time interval $[t_0, t_f]$ with width $T = t_f - t_0$.

A1) We consider that all wind generators are connected to a common bus in the power network or equivalently, that the network is uncongested, yielding uniform locational marginal prices (LMPs) across buses. All the WPPs will therefore have a common nodal price and can directly aggregate their injected power without regard to line capacity constraints.From this assumption, we will consider WPPs form a willing coalition and that they sell their aggregate power into the energy market as a single entity. We denote the aggregate power output of a coalition $\mathcal{S} \subseteq \mathcal{N}$ as :

$$w_{\mathcal{S}}(t) = \sum_{i \in S} w_i(t) \tag{1}$$

The corresponding random process is denoted by $\mathbf{w}_{\mathcal{S}} = \{w_{\mathcal{S}}(t) \leq w \mid t \in [t_0, t_f]\}$. There are $m = 2^n - 1$ possible distinct coalitions.

1.1 Market Model and Metrics

We consider the competitive power market as applied in the Netherlands. It consists of two ex-ante markets [a day-ahead (DA) forward market and a real-time (RT) spot market] and an ex-post penalty mechanism from TENNET to settle uninstructed deviations from ex-ante offers.

Let C denote the constant power contract that a coalition $\mathcal{S} \subseteq \mathcal{N}$ collectively plans to sell in a single ex-ante DA forward market over a single time interval $[t_0, t_f]$. We consider a single contract interval. We want to maximize the expected profit obtained by a coalition. The decision variable for this optimization problem is the size of the constant power contract C to be offered in the single ex-ante DA forward market. We denote the clearing price in the DA forward market by $p \in R^+$($/MWh). Deviations from a contract offered ex-ante are settled ex-post at a price $q \in R^+$ ($/MWh) for negative deviations and a price $\lambda \in R^+$ ($/MWh) for positive deviations.

A2) The WPPs behave as price takers. They will be subjected to the market prices, therefore we consider price p is assumed fixed and known.

A3) The WPPs have a zero marginal cost of production.

A4) We consider q and λ are assumed known in advance and strictly positives.

We have a set $\mathcal{N}$ of players, the Wind Power Producers. The characteristic function will here be represented by $\{d\eta(t)\}$, which is the set of all instantaneous expected profit. Our dynamic TU Game will therefore be represented as $< \mathcal{N}, \{d\eta(t)\} >$. In this game, the players are involved in a sequence of instantaneous TU games, at each time t the instantaneous TU game is $< \mathcal{N}, d\eta(t) >$. With the dynamic game we associate a dynamic average game $< \mathcal{N}, \{\bar{d\eta}(t)\} >$ and an instantaneous average game at time $t \geq 0$, $< \mathcal{N}, \bar{d\eta}(t) >$.

The core of the instantaneous game at time $t \geq 0$ is defined as followed :

$C(d\eta(t)) := \{a \in R^n \ : \sum_{i \in \mathcal{N}} a_i(t) = d\eta_m(t), \sum_{i \in \mathcal{S}} a_i(t) \geq d\eta_{\mathcal{S}}(t) \ \forall \mathcal{S} \subset \mathcal{N}\}$

The core of the average game at time $t \geq 0$ is defined as followed :

$C(\eta_{nom}) := \{a \in R^n \ : \sum_{i \in \mathcal{N}} a_i(t) = \eta^m_{nom}(t), \sum_{i \in \mathcal{S}} a_i(t) \geq \eta^{\mathcal{S}}_{nom}(t) \ \forall \mathcal{S} \subset \mathcal{N}\}$

We assume that the core of the average game is nonempty on the long run. We denote by η_{nom} the (long run) average coalitions' values, namely, $\eta_{nom} := lim_{t \to \infty} \bar{d\eta}(t)$ and let $C(\eta_{nom})$ be the core of the average game.

A5) (balancedness) The core of the average game is nonempty in the limit : $C(\eta_{nom}) \neq \emptyset$.

A6) (bounded allocation) The instantaneous allocation is bounded within a hyperbox in R^n : $a(t) \in \mathcal{A} := \{a \in R^n : a_{min} \leq a \leq a_{max}\}, a_{min}, a_{max} \in R^n$.

1.2 Stable coalition

In order to build a stable coalition, we will have to find the best allocations to every player, so that they would not leave it to build another one. This does not need to be done right away as we just need to prove that they will make more money by joining the coalition, we can focus on this part later.
Problem 1: Find an allocation rule $f : R^m \to \mathcal{A} \in R^n$, such that if $a(t) = f(\epsilon(t))$ then :
i) $lim_{t\to\infty}\bar{a}(t) \in \mathcal{A}_0 \subseteq C(\eta_{nom})$ w.p.1, and
ii) $lim_{t\to\infty}\epsilon(t) \in \sum_0 \subseteq R_+^m$ w.p.1.
with $\mathcal{A}_0$ the set where the average allocations converge to.

1.3 Optimazation model

Our goal is to maximize the profit of the coalition at time t by calculating what is the best amount of electricity we should predict the coalition is going to produce. We should therefore consider the profit $z(t)$ at time t :

$$z(t) = \begin{cases} \underset{C}{\textbf{Maximize}} & pC \\ \text{subject to :} & C = w_{\mathcal{S}}(t) \\ & C \in [0, \sum_{i\in\mathcal{S}} W_i] \end{cases} \tag{2}$$

where C is what the coalition $\mathcal{S}$ plans on producing and $w_{\mathcal{S}}(t)$ is what the coalition will actually produce at time t.
Looking at the problem, we see that our only variable C is fixed by $C = w_{\mathcal{S}}(t)$ so it is not really an optimization problem, moreover we don not know $w_{\mathcal{S}}(t)$ at the moment and still have to decide the value of C. We are thus going to relax this constraint and take into account the penalties q and λ we mentioned earlier into the profit. If we produce more than we said we would ($\iff w_{\mathcal{S}}(t) \geq C$), then we pay $\lambda * (w_{\mathcal{S}}(t) - C)$\$ of penalties. If we produce less than we said we would ($\iff w_{\mathcal{S}}(t) \leq C$), then we pay $q * (C - w_{\mathcal{S}}(t))$\$ of penalties. We obtain the following problem :

$$z(t) = \begin{cases} \underset{C}{\textbf{Maximize}} & pC - \lambda[w_{\mathcal{S}}(t) - C]^+ - q[C - w_{\mathcal{S}}(t)]^+ \\ \text{subject to :} & C \in [0, \sum_{i\in\mathcal{S}} W_i] \end{cases} \tag{3}$$

where $[a]^+ = max(0, a)$.

We obtained the right objective function to calculate our profit at time t, however to find the best solution C without knowing $w_{\mathcal{S}}(t)$ still is not possible with this problem. Therefore, instead of trying to maximize the profit at time t, we will try to maximize the expected profit. The expected profit is defined by :
$E(pC - \lambda[w_{\mathcal{S}}(t) - C]^+ - q[C - w_{\mathcal{S}}(t)]^+)$
$= pC - \lambda E([w_{\mathcal{S}}(t) - C]^+) - qE([C - w_{\mathcal{S}}(t)]^+)$
With this objective function, we can maximize our expected profit with the problem :

$$z(t) = \begin{cases} \underset{C}{\textbf{Maximize}} & pC - \lambda E([w_{\mathcal{S}}(t) - C]^+) - qE([C - w_{\mathcal{S}}(t)]^+) \\ \text{subject to :} & C \in [0, \sum_{i\in\mathcal{S}} W_i] \end{cases} \tag{4}$$

To ease the notation, we denote the expected profit of coalition S :
$g_{\mathcal{S}}(C, t) := pC - \lambda E([w_{\mathcal{S}}(t) - C]^+) - qE([C - w_{\mathcal{S}}(t)]^+)$.

1.3.1 Gradient dynamics

In order to write the gradient dynamics, we need to find the gradient of the expected profit. We can rewrite the expected functions such as :

$$
\begin{aligned}
g_{\mathcal{S}}(C,t) &= pC - \lambda \int_0^{\infty} [w_{\mathcal{S}}(t) - C]^+ f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) - q \int_0^{\infty} [C - w_{\mathcal{S}}(t)]^+ f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) \\
&= pC - \lambda \int_C^{\infty} [w_{\mathcal{S}}(t) - C] f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) - q \int_0^{C} [C - w_{\mathcal{S}}(t)] f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)
\end{aligned} \tag{5}
$$

where :

$$
f_w(w_{\mathcal{S}}(t)) = \frac{dP(w \leq w_{\mathcal{S}}(t))}{dw_{\mathcal{S}}(t)}
$$

Let's focus on the expression that defines positive deviations :

$$
\int_C^{\infty} [w_{\mathcal{S}}(t) - C] f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) := EE
$$

We will denote it EE as Expected quantity of energy Excess to ease the reading. Let's now focus on the expression regarding negative deviations :

$$
\int_0^{C} [C - w_{\mathcal{S}}(t)] f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) = C \int_0^{C} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) - \int_0^{C} w_{\mathcal{S}}(t) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)
$$

We can now rewrite :

$$
\begin{aligned}
\int_0^{C} w_{\mathcal{S}}(t) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) = \int_0^{\infty} w_{\mathcal{S}}(t) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) - \int_C^{\infty} (w_{\mathcal{S}}(t) - C) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) \\
- C \int_C^{\infty} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)
\end{aligned}
$$

In this expression, we recognize the mean of our stochastic variable $w_{\mathcal{S}}(t)$:
$\hat{w}_{\mathcal{S}}(t) := \int_0^{\infty} w_{\mathcal{S}}(t) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)$
and also EE that we defined earlier. Thus, we obtain :

$\int_0^{C} w_{\mathcal{S}}(t) f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) = \hat{w}_{\mathcal{S}}(t) - EE - C \int_C^{\infty} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)$

If we substitute the above in expression (5), we obtain :

$$
\begin{aligned}
g_{\mathcal{S}}(C,t) &= pC - \lambda EE - qC \int_0^{C} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) + q(\hat{w}_{\mathcal{S}}(t) - EE - C \int_C^{\infty} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)) \\
&= pC - (\lambda + q)EE - qC \int_0^{\infty} f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) + q\hat{w}_{\mathcal{S}}(t) \\
&= pC - (\lambda + q)EE - qC + q\hat{w}_{\mathcal{S}}(t) \\
&= (p - q)C + q\hat{w}_{\mathcal{S}}(t) - (\lambda + q)EE
\end{aligned}
$$

The Leibniz integral rule for differentiation under the integral sign, states that for an integral of the form $\int_{a(x)}^{b(x)} h(x,t)\, dt$ where $-\infty < a(x), b(x) < \infty$ the derivative of this integral is expressible as :
$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} h(x,t)\, dt\right) = h\big(x, b(x)\big) \cdot \frac{d}{dx} b(x) - h\big(x, a(x)\big) \cdot \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} h(x,t)\, dt,$$

We want to use this Leibniz's Rule to differentiate EE. To do that we will replace ∞ by a integer denoted by A that is bigger than the sum of what all the WWP $\in \mathcal{N}$ could produce and we obtain :

$$
\begin{aligned}
\frac{\partial EE}{\partial C} &= \frac{\partial}{\partial C}\left(\int_C^{A} [w_{\mathcal{S}}(t) - C] f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t)\right) \\
&= 0 - 0 + \int_C^{A} \frac{\partial}{\partial C}\left([w_{\mathcal{S}}(t) - C] f_w(w_{\mathcal{S}}(t))\right) dw_{\mathcal{S}}(t) \\
&= \int_C^{A} -f_w(w_{\mathcal{S}}(t)) dw_{\mathcal{S}}(t) \\
&= -P[w_{\mathcal{S}}(t) \geq C] \\
&= P[w_{\mathcal{S}}(t) \leq C] - 1
\end{aligned} \tag{6}
$$

Finally, we obtain the gradient of $g_{\mathcal{S}}(C,t)$:

$$\begin{aligned}\frac{\partial g_{\mathcal{S}}(C,t)}{\partial C} &= (p-q) - (\lambda+q)(P[w_{\mathcal{S}}(t) \leq C] - 1) \\ &= p - \lambda(P[w_{\mathcal{S}}(t) \leq C] - 1) - qP[w_{\mathcal{S}}(t) \leq C]\end{aligned} \tag{7}$$

Let $\eta(t) = \{\eta_1(t), \eta_2(t), ..., \eta_m(t)\}$ be the vector of the characteristic functions representing the expected profit of each coalition from time t_0 to time t, $d\eta(t)$ describes the instantaneous profit at time t. Let $g(C,t) = \{g_{\mathcal{S}}(C,t)\}_{\mathcal{S}\in\{0,1,2,..,m\}}$ the vector of the expected profits of each coalition for an expected production C at time t. The characteristic functions can be modeled as a diffusion process with drift, so its evolution can be described as :

$$\begin{cases} d\eta_{\mathcal{S}}(t) = \dfrac{\partial g_{\mathcal{S}}(C,t)}{\partial C} dt \\ \eta_{\mathcal{S}}(t_0) = \eta_{\mathcal{S},t_0} \end{cases} \tag{8}$$

If we replace the differential of $g_{\mathcal{S}}(C,t)$ by expression 7 we get :

$$\begin{cases} d\eta_{\mathcal{S}}(t) = [(p-q) - (\lambda+q)(P[w_{\mathcal{S}}(t) \leq C] - 1)]dt \\ \eta_{\mathcal{S}}(t_0) = \eta_{\mathcal{S},t_0} \end{cases} \tag{9}$$

We will denote $\tilde{b}(t) = \int\limits_{t_0}^{t} b(\tau)d\tau$ and $\bar{b}(t) = \frac{\tilde{b}(t)}{t-t_0}$ for any process {b(t)}. Thus $\bar{d\eta}(t) := \frac{\eta(t)}{t-t_0}$ represents the average value of $\eta(t)$ from t_0 to t. For the rest of the paper, we will use $t_0 = 0$ without loss of generality.

The next step is the simulation that will show that the coalition makes more profit than the sum of what all WPPs can do alone.

1.4 Simulation

References